Name: _____

Instructor: _____

**Math 10560, Exam 1**
**February 19, 2013**

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

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| 1. | (a) | (b) | (c) | (d) | (e) |
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| Multiple Choice | _____ |
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Multiple Choice

**1.**(6 pts) The function

$$f(x) = e^{(x^3)} + 3x + 1$$

is a one-to-one function (there is no need to check this). What is $(f^{-1})'(2)$?

(a) $\frac{1}{e}$ (b) $3$ (c) $\frac{1}{3}$ (d) $-1$ (e) $1$

**2.**(6 pts) Find the derivative of the function

$$f(x) = \frac{(x^2+1)\sin^2 x}{\sqrt{x^3+2}}.$$

(a) $\dfrac{2x}{x^2+1} + \dfrac{2\cos x}{\sin x} - \dfrac{3x^2}{2}$

(b) $\left(\dfrac{(x^2+1)\sin^2 x}{\sqrt{x^3+2}}\right)\left[2x + 2\sin x\cos x - \dfrac{3x^2}{2\sqrt{x^3+2}}\right]$

(c) $2x + 2\sin x\cos x - \dfrac{3x^2}{2\sqrt{x^3+2}}$

(d) $\left(\dfrac{(x^2+1)\sin^2 x}{\sqrt{x^3+2}}\right)\left[\dfrac{2x}{x^2+1} + \dfrac{2\cos x}{\sin x} - \dfrac{3x^2}{2(x^3+2)}\right]$

(e) $\left(\dfrac{(x^2+1)\sin^2 x}{\sqrt{x^3+2}}\right)\left[\dfrac{1}{x^2+1} + \dfrac{2}{\sin x} - \dfrac{1}{2(x^3+2)}\right]$

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**3.**(6 pts) Compute the integral

$$\int_0^{\ln 2} \frac{e^{3x}}{1+e^{3x}} dx.$$

(a) $\ln 2$

(b) $\frac{1}{3}(\ln 9 - \ln 2)$

(c) $3(\ln 9 - \ln 2)$

(d) $\ln 9$

(e) $\frac{1}{3}\ln 9$

**4.**(6 pts) Evaluate the derivative

$$\frac{d}{dx}(\sin x)^{(x^2+1)}$$

(a) $(\cos x)^{2x}$

(b) $\left[2x\ln(\sin x) + \frac{(x^2+1)\cos x}{\sin x}\right]$

(c) $(\sin x)^{(x^2+1)}\left[2x\ln(\sin x) + \frac{(x^2+1)\cos x}{\sin x}\right]$

(d) $(x^2+1)(\sin x)^{x^2}\cos x$

(e) $(\sin x)^{(x^2+1)}\ln(\sin x)$

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**5.**(6 pts) The population of Derivitania was 5000 on Jan 01, 2000. On Jan. 01, 2005, the population had risen to 5500. Using the exponential growth model for this population, which of the following gives an estimate for the size of the population of Derivitania on Jan. 01, 2020?

(a) $5000e^{20\ln(1.1)} = 5000(1.1)^{20}$

(b) $5000e^{20}$

(c) $5000e^{20\ln(1.2)} = 5000(1.2)^{20}$

(d) $e^{4\ln(1.2)} = (1.2)^4$

(e) $5000e^{4\ln(1.1)} = 5000(1.1)^4$

**6.**(6 pts) Evaluate the integral

$$\int_1^{e^{1/\sqrt{2}}} \frac{1}{x\sqrt{1-(\ln x)^2}}dx.$$

(a) $\dfrac{\pi}{4}$

(b) $\dfrac{\pi}{6}$

(c) $0$

(d) $\dfrac{1}{\sqrt{2}}$

(e) $\tan^{-1}\left(\dfrac{1}{\sqrt{2}}\right)$

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**7.**(6 pts) Evaluate the limit

$$\lim_{x\to\infty} \frac{(\ln x)^2}{x}.$$

(a) $2$ (b) $\infty$ (c) $-1$ (d) $-\infty$ (e) $0$

**8.**(6 pts) Evaluate the integral

$$\int x\,\sin(2x)\,dx.$$

(a) $-\dfrac{x\sin(2x)}{2}+\dfrac{\cos(2x)}{4}+C$

(b) $-\dfrac{\cos(2x)}{2}+\dfrac{\sin(2x)}{2}+C$

(c) $-x\cos(2x)+\sin(2x)+C$

(d) $-\dfrac{x\cos(2x)}{2}+\dfrac{\sin(2x)}{4}+C$

(e) $-\dfrac{x^2\cos(2x)}{4}+\dfrac{\sin(2x)}{4}+C$

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**9.**(6 pts) Evaluate the integral

$$\int \tan^{100} x \, \sec^4 x \, dx.$$

Note: One of the formulas given on the last page of the exam may help you with this problem.

(a) $\dfrac{\sec^{100} x}{100} + \dfrac{\sec^{101} x}{101} + C$

(b) $\dfrac{\tan^{101} x}{101} + \dfrac{\tan^{103} x}{103} + C$

(c) $\dfrac{\sec^{101} x}{101} + C$

(d) $\dfrac{\tan^{101} x}{101} + C$

(e) $\dfrac{\tan^{100} x}{100} + \dfrac{\tan^{101} x}{101} + C$

**10.**(6 pts) Which of the following expressions gives the correct form of the partial fraction decomposition of the function

$$f(x) = \frac{3x^2 + 2x + 1}{(x-1)(x-5)^2(x^2+1)}?$$

(a) $\dfrac{A}{x-1} + \dfrac{B}{(x-5)^2} + \dfrac{C}{x^2+1}$

(b) $\dfrac{A}{x-1} + \dfrac{B}{x-5} + \dfrac{C}{(x-5)^2} + \dfrac{D}{x^2+1}$

(c) $\dfrac{A}{x-1} + \dfrac{B}{x-5} + \dfrac{C}{(x-5)^2} + \dfrac{Dx+E}{x^2+1}$

(d) $\dfrac{A}{x-1} + \dfrac{Bx+C}{(x-5)^2} + \dfrac{Dx+E}{x^2+1}$

(e) $\dfrac{A}{x-1} + \dfrac{B}{(x-5)^2} + \dfrac{Cx+D}{x^2+1}$

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Partial Credit

You must show your work on the partial credit problems to receive credit!

**11.** (13 pts.) Evaluate the limit

$$\lim_{x \to 0^+} \big(\tan x\big)^x.$$

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**12.** (13 pts.) Compute the integral

$$\int \frac{10}{(x-3)(x^2+1)} dx \ .$$

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**13.** (14 pts.) Calculate the integral

$$\int_0^3 \frac{1}{\sqrt{x^2+9}}\, dx \ .$$

Note: One of the formulas given on the last page of the exam may help you with this problem.

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**The following is the list of useful trigonometric formulas:**

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}\big(\sin(x - y) + \sin(x + y)\big)$$

$$\sin x \sin y = \frac{1}{2}\big(\cos(x - y) - \cos(x + y)\big)$$

$$\cos x \cos y = \frac{1}{2}\big(\cos(x - y) + \cos(x + y)\big)$$

$$\int \sec \theta = \ln |\sec \theta + \tan \theta| + C$$

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